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Taxes and Turnout: When the Decisive Voter Stays at Home†

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We develop a model of political competition with endogenous turnout and endogenous platforms. Parties trade off incentivizing their supporters to vote and discouraging the supporters of the competing party from voting. We show that the latter objective is particularly pronounced for a party with an edge in the political race. Thus, an increase in political support for a party may lead to the adoption of policies favoring its opponents so as to asymmetrically demobilize them. We study the implications for the political economy of redistributive taxation. Equilibrium tax policy is typically aligned with the interest of voters who are demobilized. (*JEL* D63, D72, H23, H24)

This paper has two main contributions. First, it develops a model of political competition in which the parties' platform choices and voters' participation in elections are jointly determined in equilibrium. Second, it uses this framework for a political economy analysis of redistributive taxation. The previous literature has focused on exogenous turnout identifying conditions under which greater inequality leads to more redistribution, Meltzer and Richard (1981), or less, Bénabou (2000). With exogenous turnout, changes in the distribution of incomes among those who actually vote shift redistributive policies in the same direction: if voters get poorer, tax policies get more redistributive; if voters get richer, tax polices get less redistributive. We revisit this relationship.

Political Competition.—Most of the previous political economy literature has focused either on platform choices or on endogenous turnout. By combining the two we obtain a framework where parties face a trade-off between, on the one

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hand, appealing to as many voters as possible, and on the other hand, ensuring that these potential voters turn out to vote. A potential voter of, say, party 1 is weakly better off if party 1 wins and implements its platform. Being among those who prefer party 1 over party 2 is, however, only a necessary condition for voting in favor of party 1. Potential voters are turned into actual voters only if the stakes are sufficiently high; i.e., they must be incentivized to fight for a victory of their party. A voter who is close to being indifferent between the two parties lacks such incentives, since in this case the gain in utility from having her preferred party elected does not justify incurring the voting cost. Thus, parties face a trade-off between adopting polices that increase the size of their base and policies that foster mobilization.

We draw on the probabilistic voting model—see Coughlin and Nitzan (1981) and Lindbeck and Weibull (1987)—to determine how voters sort into the two parties' bases. Specifically, voters have both policy preferences and idiosyncratic party preferences. A voter can therefore be attracted to the base of party 1 because she likes the platform of party 1 better, or because she likes party 1 for exogenous reasons. With well-behaved distributions of these preferences, a party's base responds in a continuous way to changes in the party's platform, and there are pure strategy equilibria even with multidimensional policy spaces. The probabilistic voting model is one of the workhorses in the formal analysis of party competition. However, this literature typically assumes that voter turnout is exogenous.

We draw on models of ethical voting—originally proposed by Harsanyi (1980) and more recently analyzed by Coate and Conlin (2004) and Feddersen and Sandroni (2006)—to endogenize turnout. These models have been proposed as a way of addressing the paradox of voting.[1] It is assumed that voting is costly and that voting behavior is driven by a desire to fulfill a civic duty—formalized as a group rule-utilitarian criterion for turnout. Individuals choose a turnout rule that is optimal on the assumption that everyone with the same party preferences behaves according to the same rule.[2] Such group behavior is able to affect the outcome of the election, thus leading to nontrivial equilibrium turnout rates. These depend on how much voters have at stake: when their aggregate benefit from winning the election is higher, more individuals of a given group turn out to vote. This literature delivers predictions that are consistent with empirical facts on turnout, but it generally considers exogenous policy platforms.

Our formal analysis merges these two models so that both policies and turnout are endogenous outcomes. We focus on the implications of the trade-off between the number of potential voters and mobilization. We establish conditions for equilibrium existence, fully characterize the equilibrium analytically, and provide a comparative statics analysis.

With endogenous turnout, either party has an incentive to propose a platform that is very attractive for its own followers so that they have a good reason to vote. In

[1] The paradox is that observed turnout in elections is positive even though rational agents have no incentive to participate since the probability of being pivotal in large elections is negligible.

[2] Ethical voter models differ in some aspects: for instance, Feddersen and Sandroni (2006) model the electorate as being split between ethical and nonethical voters. Coate and Conlin (2004) only have ethical voters in their framework. Our analysis is closer to Feddersen and Sandroni (2006), but we could as well have adopted the modeling choices of Coate and Conlin (2004). We provide a more detailed comparison of these approaches in the online Appendix, where we also show that these modeling choices are inconsequential for our main results.

addition, there is also a demobilization objective for the followers of the competing party. This generates a countervailing incentive to propose a platform that is, from their perspective, as good as the platform proposed by their own party, so that they may as well stay at home on election day. A main finding of our comparative statics analysis is that this demobilization objective gets more weight for a party with strong support from its potential voters. By contrast, a party that only has lukewarm support from its followers, and is thus unlikely to win the election, should put more weight on the mobilization of its own base. The underlying mechanism is a differential elasticity effect: The enthusiasm of the stronger party's supporters makes them less responsive to the proposed policies. The more modest supporters of the competing party show a stronger response to changes in the parties' platforms. As a consequence the electoral returns from a platform that caters to them are larger.

Campaign strategies where a front-runner avoids controversial positions or even adopts positions of the rival are also referred to as strategies of *asymmetric demobilization.* Empirical evidence for this mechanism is given by Chen (2013, p.1), who shows in the context of hurricane disaster aid in Florida in 2002 that "an incumbent who delivers distributive benefits to the opposing party's voters partially mitigates these voters' ideological opposition to the incumbent, hence weakening their motivation to turn out and oust the incumbent." Another prominent example are the campaigns of the Christian Democratic Union of Germany (CDU) in the era of Angela Merkel; see, e.g., Schmidt (2014).[3] Our analysis sheds light on the strategic considerations that rationalize such a strategy. Moreover, in online Appendix E we present a detailed case study of the federal elections in Germany between 2005 and 2017 and argue that the empirical outcomes are aligned with the comparative statics predictions of our model.

Redistributive Taxation.—A classic hypothesis in the political economy of taxation is that increased inequality leads to more redistributive taxation. This hypothesis is usually derived from a model with a *decisive voter* who has below-average income, see Meltzer and Richard (1981).[4] Meltzer and Richard argue moreover that, historically, extensions of the franchise added voters with below-average income and thus reduced the income of the decisive voter. Bénabou (2000, p.107) documents that "*every reported form of political activity rises with income and education.*" Assuming that turnout is, for exogenous reasons, larger among "the rich," Bénabou (2000) presents an analysis of redistributive taxation that is based on the assumption that there is a decisive voter with an above-median income.[5]

Both Meltzer and Richard (1981) and Bénabou (2000) exemplify a common and natural perspective on the political economy of redistributive taxation. First, there is a decisive or pivotal voter, defined as the voter whose preferred policy coincides with the policy implemented in a political equilibrium. Second, changes in who participates in elections and changes in the preferences of the decisive voter go together. If the electorate becomes poorer due the extension of the franchise, then

[3] It seems that the term *asymmetric demobilization* had its first appearance in an analysis of a regional election in Catalonia; see Lago, Montero, and Torcal (2007).

[4] In their framework, under universal suffrage and majority rule, the decisive voter is the voter with median income.

[5] The framework in Bénabou (2000) not only has efficiency costs due to the behavioral responses to taxation, but there also is scope for efficiency gains from redistributive taxation due to market incompleteness.

the decisive voter becomes also poorer and demands for redistributive taxation go up. If incomes among those who actually vote or otherwise participate in the political process are higher than incomes of those with the right to vote, then the decisive voter is richer, and demands for redistribution are more limited—as compared to a situation with universal or even turnout.

Our analysis of redistributive taxation with endogenous turnout gives rise to a different logic. The *decisive voter* and *turnout of "the rich" relative to "the poor"* may change in different directions when the political environment changes.[6] In particular, the decisive voter may become poorer when turnout gets larger among "the rich" and smaller among "the poor." This is an implication of the asymmetric demobilization logic. To illustrate this result, suppose that there is a race between a pro-market party and a more left-leaning party. The pro-market party gets more support from richer voters who are also more opposed to redistribution. The left-leaning party gets more support from poorer voters who benefit if redistributive taxes go up. Let there be an initial situation that is balanced, i.e., where both parties are equally likely to win the election and neither party has a turnout advantage. Now suppose that, for exogenous reasons, the supporters of the pro-market party become more willing to fight for a victory of their party—in the model, a shock that raises the intensity of their party preferences—then, in the resulting new equilibrium, the pro-market party has a turnout advantage, with the implication that turnout gets larger among "the rich" than among "the poor." The pro-market party is now more likely to win the election and the demobilization objective gains in importance. It therefore adopts a more redistributive platform and equilibrium taxes go up: the decisive voter gets poorer. Overall, the supporters of the pro-market are still better off. The benefits from an increased probability of winning outweigh the losses from a more redistributive policy.

This finding is shown to be robust in a variety of dimensions. For instance, it does not depend on which model of redistributive taxation is used. It holds for a model with affine income taxes, tax schedules with a constant rate of progressivity, or Mirrleesian nonlinear income tax schedules. It also holds for a broad class of comparative statics experiments which all imply that a pro-market party gains strength over a more left-leaning competitor. As a response, the pro-market party increasingly seeks to demobilize the supporters of the more left-leaning party by adopting a more redistributive platform.

Related Literature.—Our analysis relates to the literature that seeks a response to the paradox of voting. We draw on one strand of this literature, models of ethical voting, due to Harsanyi (1980), Coate and Conlin (2004), and Feddersen and Sandroni (2006); see also Feddersen (2004) for a survey, and Callander and Wilson (2007), Degan and Merlo (2011), Aldashev (2015), or Alger and Laslier (2021) for more recent contributions. Rational voting (see, e.g., Ledyard 1984), is a prominent

[6] In our framework, voters differ in their position in the income distribution and in their party preferences. For simplicity, and to connect with the previous literature, we refer to "the decisive voter" as the one whose ideal tax policy is implemented in equilibrium. With a conventional probabilistic voting model, preferences over tax policies depend only on the position in the income distribution. Hence, there is no need to look into the decisive voter's party preferences. A difference to a Downsian framework in which party preferences play no role is, moreover, that the decisive voter is not generally the one with median income among those who turn out to vote.

alternative to ethical voting. Coate, Conlin, and Moro (2008) argue that the ethical voter model provides a better fit for data on turnout than the pivotal voter model.

We contribute to a rich literature on the political economy of redistributive taxation. Different models of redistributive taxation are employed by this literature. For instance, Roberts (1977) and Meltzer and Richard (1981) use a model of linear income taxation. Bénabou (2000) considers tax schedules with a constant rate of progressivity; see Heathcote, Storesletten, and Violante (2017) for a detailed analysis of such tax schedules in dynamic settings. Our analysis of redistributive taxation applies both to linear income taxes and to tax schedules with a constant rate of progressivity. It also applies to fully nonlinear income taxes. Political economy treatments of nonlinear taxation have been provided by, e.g., Fahri and Werning (2008); Acemoglu, Golosov, and Tsyvinski (2008, 2010); Brett and Weymark (2017); or Bierbrauer and Boyer (2016).

Different turnout rates among "the rich" and "the poor" are frequently discussed as a potential explanation for limited redistribution; see, e.g., Bénabou (2000), Larcinese (2007), Sabet (2016), and the references therein. This literature treats turnout as an exogenous variable; i.e., the possibility that turnout may depend on the parties' policy proposals has not been taken into account.

There is a rich literature in political science that investigates to what extent parties cater towards their core voters or to swing voters; Cox (2010) provides a survey. It has been shown empirically that parties may also have an incentive to target their promises to the core voters of the competing party to mitigate their turnout; see Chen (2013). We contribute to this literature by developing a tractable theoretical framework that rationalizes these competing effects, and by finding that the incentive for demobilization is stronger for a party that is the likely winner of an election. Bernhardt, Buisseret, and Hidir (2018) derive a similar result, albeit from a model with exogenous turnout. Adams and Merrill (2003) set up a different model of endogenous turnout, based on individual abstention thresholds rather than group behavior. Their framework also leads to the finding that candidates may appeal to their base of core supporters as well as to their rival's base of supporters. Our model, however, provides an explicit microfoundation of voting behavior to deal with the paradox of voting, based on Coate and Conlin (2004) and Feddersen and Sandroni (2006). Moreover, it allows us to derive a sharp characterization of the relative weights that the parties' objectives attach to each group of voters (swing voters, own core voters, and rival core voters) in equilibrium.

Outline.—The remainder of the paper is organized as follows. Section I introduces a general setup for an analysis of political competition that connects probabilistic voting with endogenous turnout. In Section II we clarify what this framework implies for the political economy of redistributive taxation. We provide conditions for equilibrium existence in Section III. Proofs of propositions and of all other formal statements in the paper are in the online Appendix.

I. Party Competition with Endogenous Turnout

Two political parties $j \in \{1,2\}$ compete by choosing policies from a set of feasible policies $\mathcal{P}$. Party j's proposal is denoted by $p^j \in \mathcal{P}$. The policy space $\mathcal{P}$ can be one-dimensional or multidimensional.

Preferences.—There is a continuum of citizens of mass one. Citizens differ in their preferences over policies. For any $\omega \in \Omega$, we denote by $u(p,\omega)$ the utility that a type-ω citizen realizes under policy $p \in \mathcal{P}$. In the income tax application, ω will determine an individual's position in the income distribution and will thus shape preferences over redistributive taxation.[7] The cross-sectional distribution of types $\omega \in \Omega$ is common knowledge and represented by a cumulative distribution function F_ω with density f_ω.

Individuals also have party preferences. These preferences may be shaped by cultural and ethnic identities, party histories, or fixed party positions in certain policy domains. Formally, the random variable $\varepsilon \in \mathbb{R}$ denotes an agent's idiosyncratic preference for party 2. Conditional on ω, party preferences ε of different voters are independent and identically distributed. Thus, an individual with type ω and party preference ε supports party 1 if

$$u(p^1,\omega) - u(p^2,\omega) \geq \varepsilon.$$

We denote by $B(\cdot \mid \omega)$ the cumulative distribution function of party preferences ε among individuals of type ω, and by $b(\cdot \mid \omega)$ the corresponding density function. Therefore, the fraction of type-ω individuals supporting party 1 is $B\big(u(p^1,\omega) - u(p^2,\omega) \mid \omega\big)$.

Ethical Voting.—The mass of type-ω supporters of each party j is split into two groups: a fraction $1 - \tilde{q}^j(\omega)$ of these agents always abstains from voting, whereas the complementary fraction $\tilde{q}^j(\omega)$ consists of the *ethical voters*.[8] These voters turn out to vote according to a group rule-utilitarian calculation. This calculation yields a strategy for turnout that depends on the parties' platforms and on voting costs. The rule-utilitarian aspect is that—rather than free riding on the turnout of others—ethical voters behave according to the strategy that maximizes utilitarian welfare on the assumption that this strategy is followed also by all other ethical voters. The group aspect is that they take account only of people who share their party preference when computing utilitarian welfare. Thus, an ethical supporter of party 1 turns out to vote if the optimal strategy for the supporters of party 1 says that he or she should do so. Below, we provide a formal characterization of the optimal strategy for ethical voters.

We seek a framework where the election outcome is uncertain both from the voters' and the parties' perspectives. A convenient approach, adopted by Feddersen and Sandroni (2006), is to assume that $\tilde{q}^j(\omega)$ is a random variable and that its realization is unknown both when parties choose platforms and when potential voters decide whether or not to turn out. More specifically, we assume that $\tilde{q}^1(\omega)$ and $\tilde{q}^2(\omega)$ have the same expected value $\bar{q}(\omega) \in (0,1)$; that is, a type-$\omega$ supporter of party 1 is, on average, as likely to be of the ethical type as a type-ω supporter of party 2. The following assumption puts structure on how realizations of $\tilde{q}^1(\omega)$ and $\tilde{q}^2(\omega)$ relate to the mean.

[7] The online Appendix contains an application where ω represents public goods preferences.

[8] We follow Coate and Conlin (2004) and Feddersen and Sandroni (2006) and assume that there are no "always-voters," i.e., individuals who come to the ballot regardless of how high their voting costs are. In the online Appendix, we present a version of our model that includes such voters and gives rise to an equilibrium analysis that is equivalent to the one developed in the body of the text.

ASSUMPTION 1. *For each party j, there is a nonnegative random variable η^j with mean one such that $\tilde{q}^j(\omega) = \eta^j \cdot \bar{q}(\omega)$, for all $\omega \in \Omega$.*

The possibility that party 1 is affected by a positive shock $\eta^1 > 1$ and party 2 is affected by a negative shock $\eta^2 < 1$, or vice versa, generates uncertainty in election outcomes. The random variable η^j can be interpreted as capturing the success of an election campaign that is revealed only on election day.[9] Assumption 1 is imposed in the sequel without further mention.

Bases.—The ethical supporters are a party's potential voters. For ease of exposition, we also refer to the expected mass of these agents as a party's *base*; that is, given two policies p^1 and p^2, the base of party 1 is given by

$$\mathbf{B}^1(p^1, p^2) = E\Big[\bar{q}(\omega)B\big(u(p^1,\omega) - u(p^2,\omega)\,|\,\omega\big)\Big], \tag{1}$$

where the expectation operator E indicates the computation of a population average with respect to different types ω. We define the base of party 2, $\mathbf{B}^2(p^1, p^2)$, analogously.

Stakes.—The *stakes* for the potential voters of party 1 are defined as the expected (utilitarian) welfare gain that is realized if a victory by party 2 is avoided. Formally,

$$W^1(p^1, p^2) = E\Big[\int_{\mathbb{R}} \max\big\{u(p^1,\omega) - u(p^2,\omega) - \varepsilon,\, 0\big\} b(\varepsilon\,|\,\omega) d\varepsilon\Big]. \tag{2}$$

The integrand in equation (2) is the difference in utilities realized under the policies p^1 and p^2, including the gains or losses due to party preferences. The *max* indicates that the summation over ε takes into account only the agents for whom this utility difference is positive, i.e., the supporters of party 1. We define $W^2(p^1, p^2)$ analogously.

Voting Costs.—We denote by σ^j the fraction of ethical supporters of party *j* who actually turn out to vote. We define the aggregate voting cost of the ethical supporters of party *j* by $\kappa(\sigma^j)\,\mathbf{B}^j(p^1, p^2)$, where, for some scalars $\chi > 0$ and $\lambda \in (0,1]$,

$$\kappa(\sigma^j) = \chi\,(\sigma^j)^{1/\lambda}. \tag{3}$$

This isoelastic functional form unifies several cases. First, suppose that all the ethical voters have a common per capita voting cost χ and choose an individual probability of voting σ^j.[10] We then obtain a linear voting cost function $\kappa(\sigma^j) = \chi\sigma^j$, corresponding to $\lambda = 1$ in (3). Second, as will become clear below, the limit case $\lambda \to 0$ turns our setup into a standard probabilistic voting model with

[9] Note that the shocks to the two parties' bases may be correlated. We do not impose an assumption of independence.

[10] With an appeal to a law of large numbers, such a probability can also be interpreted as the percentage share of ethical voters who actually turn out to vote.

exogenous turnout. Third, our framework nests the case of quadratic voting costs as in Coate and Conlin (2004) and Feddersen and Sandroni (2006), if $\lambda = 1/2$.[11]

Endogenous Turnout.—The ethical supporters of each party j adhere to a rule σ^j for participation in the election that maximizes their aggregate expected utility, taking the costs of voting into account. As a consequence, turnout depends on the parties' policy proposals. Specifically, the problem of the ethical supporters of party j admits the following representation:[12] Taking as given the policies (p^1, p^2), and the other party's turnout rule σ^{-j}, choose $\sigma^j \in [0,1]$ to maximize

$$\pi^j(p^1,p^2,\sigma^1,\sigma^2)\, W^j(p^1,p^2) - \kappa(\sigma^j)\, \mathbf{B}^j(p^1,p^2), \tag{4}$$

where π^j is the probability that party j wins the election. This problem involves a trade-off between the probability of winning and the costs of voting, as both π^j and $\kappa(\sigma^j)\,\mathbf{B}^j$ are increasing in σ^j.

Given p^1 and p^2, an equilibrium of the *turnout game* is a pair of turnout rates $(\sigma^{1*}(p^1,p^2), \sigma^{2*}(p^1,p^2))$ that are mutually best responses. We are interested in equilibria that are interior, i.e., such that turnout responds at the margin to changes in proposed policies. Corner solutions where all or none of the ethical voters participate are conceivable. In this case, turnout is locally irresponsive to the policies that the parties propose, with the implication that the parties' strategic considerations are as in a model with exogenous turnout. In what follows, we assume interior turnout rates.[13]

A. *Base versus Turnout*

Parties face a trade-off between adopting polices that enlarge the size of their base and adopting policies that increases the turnout of their own supporters relative to the supporters of the competing party. In this section, we elaborate on this trade-off. Let

$$\bar{\pi}^1(p^1,p^2) := \pi^1\big(p^1,p^2,\sigma^{1*}(p^1,p^2),\sigma^{2*}(p^1,p^2)\big)$$

denote party 1's probability of winning, taking into account that policy choices affect the equilibrium of the turnout game. A pair of *equilibrium policies* (p^1, p^2)

[11] These papers derive the quadratic cost function from a setup in which individual voting costs are i.i.d. draws from a uniform distribution, and a cutoff rule σ^j so that all individuals with voting costs below that threshold turn out to vote.

[12] A derivation can be found in the online Appendix. It is based on an analysis of expected welfare. We show that expected welfare of the supporters of party 1, say, can be written as a sum that involves only two relevant terms, one which gives the expected welfare gain in case their party wins and one which captures the costs of voting. The first term arises because expected welfare of party 1 supporters is equal to π^1 times their welfare under policy p^1, plus $(1 - \pi^1)$ times their welfare under policy p^2. Maximizing this expression with respect to π^1 is equivalent to maximizing π^1 times the difference in welfare between policies p^1 and p^2, that is, $\pi^1 \times W^1$.

[13] Inada conditions ensure that the equilibrium of the turnout game is interior: the cumulative distribution function (c.d.f.) F_η of the random variable η^2/η^1 is concave with a bounded and strictly positive density f_η, and the cost function κ is convex with $\lim_{\sigma\to 0} \kappa'(\sigma) = 0$ and $\lim_{\sigma\to 1} \kappa'(\sigma) = \infty$. Alternative conditions are conceivable; e.g., we could also impose Inada-like conditions on the function F_η to ensure an interior equilibrium when the cost function is not as convex.

satisfies $\bar{\pi}^1(p^1,p^2) \geq \bar{\pi}^1(\hat{p}^1,p^2)$, for all $\hat{p}^1 \in \mathcal{P}$ and $\bar{\pi}^1(p^1,p^2) \leq \bar{\pi}^1(p^1,\hat{p}^2)$, for all $\hat{p}^2 \in \mathcal{P}$.

As an implication of Assumption 1, maximizing the probability of winning for party 1 is equivalent to maximizing

$$\frac{\mathbf{B}^1(p^1,p^2)}{\mathbf{B}^2(p^1,p^2)} \times \frac{\sigma^{1*}(p^1,p^2)}{\sigma^{2*}(p^1,p^2)}, \tag{5}$$

whereas party 2's objective is to minimize this expression. The first term in (5) is a measure of the party's relative *base advantage*. The second is a measure of its relative *turnout advantage*. If turnout was exogenous, party 1 would simply focus on maximizing its base advantage, or equivalently[14] its own base $\mathbf{B}^1(p^1,p^2)$. If instead the base was exogenously given, party 1 would maximize its turnout advantage. With both an endogenous base and endogenous turnout, party 1 faces a trade-off between maximizing the number of its supporters and maximizing their relative propensity to actually vote. The turnout advantage is an endogenous object though. In the online Appendix, we provide an analysis of the ethical voters' optimization problems and explicitly solve for it. When substituting the resulting expression into equation (5), we obtain the following proposition. It shows that the parties' winning probabilities ultimately depend on the sizes of their bases and the stakes of their supporters.

PROPOSITION 1 (Base versus Stakes): *Party* 1 *maximizes and party* 2 *minimizes*

$$\Pi^1(p^1,p^2) := (1-\lambda)\ln\left(\frac{\mathbf{B}^1(p^1,p^2)}{\mathbf{B}^2(p^1,p^2)}\right) + \lambda\ln\left(\frac{W^1(p^1,p^2)}{W^2(p^1,p^2)}\right). \tag{6}$$

According to Proposition 1, with endogenous turnout, maximizing the probability of winning an election is equivalent to maximizing a weighted average of two terms, a first term that measures relative support in the population at large, and a second term that measures the party's advantage or disadvantage in the stakes that its supporters have in the election. Exogenous turnout is nested as a special case: for $\lambda \to 0$, party 1 focuses on maximizing its base $\mathbf{B}^1$.

The payoff relevance of the stake advantage when turnout is endogenous ($\lambda > 0$) implies that parties face a trade-off between mobilizing their own supporters and demobilizing the supporters of the competing party.[15] On the one hand, party 1 would like to propose a policy p^1 that makes W^1 as large as possible, i.e., that makes its own supporters as well off as possible compared to the welfare they would obtain under the opposition's platform p^2. Doing so encourages its own supporters to turn out by increasing how much they have at stake in the election. On the other hand, party 1 would also like to propose a policy p^1 that makes W^2 as small

[14] It is straightforward to show that the two parties' bases add up to the constant: $\mathbf{B}^2(p^1,p^2) = E[\bar{q}(\omega)] - \mathbf{B}^1(p^1,p^2)$. Hence, a change in policies that increases, say, the base of party 1, translates one-for-one into a decrease of party 2's base.

[15] The polar case $\lambda = 1$ corresponds to linear voting costs. In this case, an increase of the base translates one-for-one into additional voting costs, so that the probability of winning only depends on what an average voter has at stake, and not on how numerous potential voters are. Thus, party 1's objective is to maximize its stake advantage, regardless of the implied size of its base.

as possible, i.e., that does not hurt party 2's supporters too much compared to the welfare they could obtain under *their* preferred policy p^2. Doing so discourages the opposition from turning out by lowering how much they have at stake. Below, when discussing comparative statics, we argue that, in equilibrium, the weight on the demobilization objective is larger for a party that has a larger probability of winning the election.

B. *Equilibrium Policies*

The next proposition provides a characterization of equilibrium policies. Assume that the following regularity condition holds. *For every $p^2 \in \mathcal{P}$ there exists a unique solution to* $\max_{p^1} \Pi^1(p^1,p^2)$, *and for every $p^1 \in \mathcal{P}$ there exists a unique solution to* $\min_{p^2} \Pi^1(p^1,p^2)$. *The relationship between best responses and solutions to first-order conditions is one-to-one.* Under this condition, the analysis of equilibrium policies can focus on first-order conditions.[16]

Henceforth, we use shorthand expressions for the main variables of our model when both parties propose the same policy $p^1 = p^2$. Specifically, we denote the mass of type-ω citizens supporting party 1 by $B^s(\omega) := B(0\,|\,\omega)$, and the fraction of agents who are on the verge of indifference between the two parties by $b^s(\omega) := b(0\,|\,\omega)$. Furthermore, for $j \in \{1,2\}$ we denote party j's aggregate base $\mathbf{B}^j(p^1,p^2)$ and stakes $W^j(p^1,p^2)$ when $p^1 = p^2$ by $\mathbf{B}^{js}$ and W^{js}, respectively. The sorting of ethical voters into the parties' bases is then entirely driven by idiosyncratic party preferences $\varepsilon \lesseqgtr 0$. Therefore, the variables $\mathbf{B}^{js}, W^{js}$ are exogenous primitives of the model akin to moments of the distributions $B(\cdot \mid \omega)$, $\omega \in \Omega$. In particular, we have

$$(7) \qquad W^{1s} = E\left[\int_{-\infty}^{0} |\varepsilon| dB(\varepsilon \mid \omega)\right], \text{ and } W^{2s} = E\left[\int_{0}^{\infty} \varepsilon dB(\varepsilon \mid \omega)\right].$$

The variables W^{1s}, W^{2s} are thus respectively equal to the average values of the political biases of supporters of party 1 (for whom $\varepsilon < 0$) and party 2 (for whom $\varepsilon > 0$). These two *preference intensity* parameters play an important role in the sequel.

PROPOSITION 2 (Political Equilibrium): *There is a unique pure strategy equilibrium. This equilibrium is symmetric. The equilibrium policy solves*

$$\max_{p \in \mathcal{P}} E\left[\gamma^*(\omega) u(p,\omega)\right], \textit{with} \quad \gamma^*(\omega) = (1-\lambda)\,\gamma_B^*(\omega) + \lambda\,\gamma_S^*(\omega),$$

[16] It holds, for instance, if for every p^2 the function $p^1 \mapsto \Pi^1(p^1,p^2)$ is globally concave, and for every p^1 the function $p^2 \mapsto \Pi^1(p^1,p^2)$ is globally convex. In this case, there is a pure strategy equilibrium, the saddle point of the function Π^1. In Section III, we give an example of conditions on the primitives of the model—the functions $B(\cdot \mid \omega), \omega \in \Omega$ that govern the joint distribution of party and policy preferences—which imply that this regularity condition is satisfied. We also provide sufficient conditions for the existence of pure and mixed strategy equilibria beyond this special case.

where $\gamma_B^*(\cdot)$ *and* $\gamma_S^*(\cdot)$ *are given by*

$$(8)\quad \gamma_B^*(\omega) = \frac{E[\bar{q}]}{\mathbf{B}^{1s}\mathbf{B}^{2s}}\bar{q}(\omega)\, b^s(\omega) \text{ and } \gamma_S^*(\omega) = \frac{1}{W^{1s}}B^s(\omega) + \frac{1}{W^{2s}}\big(1 - B^s(\omega)\big).$$

According to Proposition 2, both parties propose the policy that maximizes a weighted utilitarian welfare function. The weights reflect that parties choose platforms so as to strike a balance between two considerations: the benefit of enlarging their set of potential supporters and the benefit of having a better turnout margin. More formally, the political equilibrium weight of type ω, $\gamma^*(\omega)$, is an average of two weights. The first one, $\gamma_B^*(\omega)$, reflects a party's gain from enlarging its base. The ratio

$$\frac{\gamma_B^*(\omega)}{\gamma_B^*(\omega')} = \frac{\bar{q}(\omega)\, b^s(\omega)}{\bar{q}(\omega')\, b^s(\omega')}$$

can be interpreted as a marginal rate of substitution that measures a party's willingness to trade-off favors to voter types ω and ω' when the size of the base is all that matters. This expression highlights the benefits of catering to swing voters that are familiar from probabilistic voting models.

The second weight, $\gamma_S^*(\omega)$, captures the contribution of the stakes margin to the probability of winning. The corresponding trade-off between the interests of different voters can be understood by considering the ratio

$$(9)\qquad \frac{\gamma_S^*(\omega)}{\gamma_S^*(\omega')} = \frac{B^s(\omega) + \frac{W^{1s}}{W^{2s}}\big(1 - B^s(\omega)\big)}{B^s(\omega') + \frac{W^{1s}}{W^{2s}}\big(1 - B^s(\omega')\big)}.$$

From the perspective of party 1, the terms $B^s(\omega)$ and $B^s(\omega')$ stem from the incentives to increase the stakes of voters who belong to its own base, whereas $1 - B^s(\omega)$ and $1 - B^s(\omega')$ reflect the benefits of reducing the stakes for voters who belong to the base of party 2. These terms reflect party attachments, rather than propensities to swing vote. Moreover, W^{1s}/W^{2s} is the ratio of party preference intensities among the supporters of parties 1 and party 2, conditional on $p^1 = p^2$.

Asymmetric Demobilization: The weights $\gamma_S^*(\omega)$ point to the returns from mobilizing the party's own base and those from demobilizing the opponent's base. Their respective contributions to the overall objective of winning the election depend, moreover, on the ratio W^{1s}/W^{2s}. From the perspective of party 1, the larger this ratio is, the more important the demobilization objective relative to the mobilization objective is. This observation gives rise to a relationship between a party's likelihood of winning the election and its incentive to demobilize the supporters of its rival—by adopting a platform closer to their preferred policy. When W^{1s} goes up and/or W^{2s} goes down, party 1's equilibrium probability of winning goes up (see equation (6)), and so does the party's incentive to cater to ω-types where party 2 has a large mass of voters in its base (see equation (9)). Proposition 3 below contains a formal statement of this insight and, moreover, clarifies its implications for redistributive taxation.

Intuitively, the underlying force is a differential elasticity effect, as we discussed in the introduction. Consider party 1: When maximizing (6), the marginal effect of a change in the proposed policy on $W^1(p^1,p^2)$ equals $\lambda W_1^1(p^1,p^2)/W^1(p^1,p^2)$ where $W_1^1(p^1,p^2)$ is a shorthand for the derivative of W^1 with respect to the first argument. Thus, the marginal effect is driven by the semielasticity $W_1^1(p^1,p^2)/W^1(p^1,p^2)$ that describes how the stakes of its supporters respond to a platform change. The semielasticity that captures the effect on the competing party's supporters equals $W_2^1(p^1,p^2)/W^2(p^1,p^2)$. When W^{1s} goes up and/or W^{2s} goes down then, in equilibrium, the first semielasticity goes down and the second one goes up. Both parties respond by adopting a policy that is more in line with what the more elastic voters want.

The campaigns of the Christian democrats (CDU) in Germany in the era of Angela Merkel are a prominent empirical example of an *asymmetric demobilization* strategy. Asymmetric demobilization was adopted in the 2009, 2013, and 2017 elections in response to the 2005 experience, in which Merkel ran on a pro-market platform and almost lost despite a significant lead in the polls over the main competitor, the Social Democrats (SPD). In subsequent elections the CDU adopted many positions previously held only by the SPD.[17] In online Appendix E we present a detailed case study of the federal elections in Germany between 2005 and 2017 and argue that outcomes in various dimensions—margin of victory, overall turnout, relative turnout for the incumbent and the challenger, economic policy orientation—are aligned with the comparative statics predictions of our model. Specifically, we use quantitative measures of party positions, to document that, after the 2005 election, the CDU moved closer to the SPD. We also document that the CDU's margin of victory over the SPD increased from 1 percent in 2005 to more than 10 percent in 2009 and the subsequent elections. Moreover, in the 2009, 2013, and 2017 elections, overall turnout was lower than in all previous elections since WWII. Crucially, however, turnout was lower among potential SPD voters than among potential CDU voters, see Jung, Schroth, and Wolf (2010); Forschungsgruppe Wahlen (2013a, b, 2015, 2018). To give a specific example, in 2009 overall turnout went down by 6.9 percentage points compared to the 2005 election. This reduction hit the SPD harder than the CDU: only 52 percent of the potential SPD voters ended up voting for the SPD, whereas 62 percent of the potential CDU voters ultimately voted for the CDU.

II. Taxes

Models of redistributive taxation have in common that policy preferences are derived from a framework with the following properties. Individuals value consumption, or after-tax income, c. The generation of earnings, or pretax income, y, requires costly effort. The parameter ω captures individual heterogeneity in effort costs: workers with low (resp., high) effort costs choose high levels of earnings and therefore end up being "rich" (resp., "poor"). For reasons of tractability, it is

[17] Josef Joffe, a well-known German journalist, summarized the CDU's strategy in colorful language: "*Ms Merkel's plan is to lull the other side; don't rile them and win by keeping them at home. How did she do it after the near-disaster of* 2005*? By shifting to the left. An apostle of free markets and low taxes ten years ago, Merkel simply outflanked the left on the left … She is the best Social Democrat the SPD could have asked for*" ("Merkel Will Do What She Has to Get the Vote," *Financial Times*, August 5, 2013).

common to assume that preferences are additively separable between consumption utility $v(c)$ and effort costs $k(y,\omega)$. The effort cost function k is decreasing in y, and has a nonnegative cross-derivative k_{12}, so that the marginal effort costs of higher types are lower.[18] Frequent assumptions are that the consumption utility v is linear or logarithmic, and that the cost function is isoelastic, $k(y,\omega) = \frac{1}{1+1/e}\left(\frac{y}{\omega}\right)^{1+1/e}$ with $e > 0$.

The policy instruments under consideration are classes of tax functions $T:y \mapsto T(y)$ that specify tax payments as a function of earnings. Thus, consumption is given by $c = y - T(y)$. A redistributive tax system typically has negative tax payments for low values of y. Tax systems have to satisfy a government budget constraint so that the transfers received by "the poor" are financed by the positive taxes paid by "the rich." The policy preferences of a type-ω individual are therefore captured by

$$u(T,\omega)=\max_y\left\{v(y - T(y)) - k(y,\omega)\right\}.$$

Models of redistributive taxation differ in the classes of tax functions that they consider. In the following, we discuss three classes that are prominent in the literature: affine tax schedules, nonlinear tax schedules with a constant rate of progressivity, and the full class of all nonlinear tax schedules. We first provide a characterization of political equilibrium taxes for all these models, presuming that the regularity conditions described in Section IB are satisfied. Section IIB then contains comparative statics results on what changes in economic or political inequality imply for taxes and turnout. While the models of redistributive taxation differ in many aspects, we can provide a unified analysis: we identify forces that lead to more or less redistributive taxes and which apply in all these models.[19]

A. *Models of Redistributive Taxation*

Linear Income Taxation.—The affine income tax, introduced by Sheshinski (1972), is frequently employed for political economy analyses under the assumption of exogenous turnout.[20] In this model, the tax function takes the form $T(y) = \tau y - r$, where τ is the constant tax rate and r is a uniform lump-sum transfer. Consequently, marginal tax rates are the same for all levels of income, whereas average tax rates increase with income. Assuming quasi-linear in consumption preferences, $v(c) = c$, leads to utility-maximizing earnings that depend on τ and ω but not on r, that is, $y^* = y^*(\tau,\omega)$. Via the government budget constraint, transfers are a function of τ, $r(\tau) = \tau E\left[y^*(\tau,\omega)\right]$. Therefore, policy preferences can be represented by

$$u(\tau,\omega) = r(\tau) + (1-\tau)\,y^*(\tau,\omega) - k\left(y^*(\tau,\omega),\omega\right). \tag{10}$$

[18] It is common to interpret ω as an hourly wage, or, more generally, as a measure of individual productivity. More productive individuals need less time to generate a given earnings level and thus have lower effort costs.

[19] For ease of exposition, we do not consider idiosyncratic income risks. In models with incomplete markets, a redistributive income tax system may be desirable as an insurance device against such risks; see Bénabou (2000). The characterization of political equilibrium weights in Proposition 2 could also be used to characterize political equilibrium taxes in such an extended framework.

[20] Well-known references include Roberts (1977), Meltzer and Richard (1981), or Alesina and Angeletos (2005).

For later reference, we note that policy preferences in this model satisfy a single-crossing property according to which higher types or, equivalently, richer taxpayers benefit less from an increase in the tax rate.[21] This implies that the ideal tax rate of richer agents is lower than the ideal tax rate of poorer ones.[22]

Applying the characterization of equilibrium policies in Proposition 2 to the linear income tax model leads to a characterization of the equilibrium tax rate τ^*. Specifically, with isoelastic effort costs and quasi-linearity in consumption, τ^* satisfies

$$\frac{\tau^*}{1-\tau^*} = -\frac{1}{e}\operatorname{Cov}\left(\frac{\gamma^*(\omega)}{E[\gamma^*]}, \frac{\omega^{1+e}}{E[\omega^{1+e}]}\right), \tag{11}$$

where the weights $\gamma^*(\cdot)$ are given by (8). The left-hand side of this equation is an increasing function of τ^*. Thus, the equilibrium tax rate depends on the covariance between political weights and productive abilities. The equilibrium tax policy is closer to the ideal of those with high political weights: the more the weights are concentrated on "the poor," the higher the equilibrium tax rate is; conversely, the more they are concentrated on "the rich," the lower the tax rate is.

The equilibrium tax rate is, moreover, decreasing in the elasticity e which is a measure of the efficiency costs due to the behavioral responses to taxation. This inverse elasticity logic is familiar from optimal tax theory. It also applies to fully nonlinear taxes or those with a constant rate of progressivity, as we will now show.

Constant Rate of Progressivity.—Tax schedules with a constant rate of progressivity, *CRP* schedules for short, allow significant tractability in a variety of redistributive taxation problems. Gans and Smart (1996) and Bénabou (2000) study voting equilibria with such taxes, assuming exogenous turnout. In this setup, the tax function is given by $T(y) = y - ry^{1-\tau}$, so that both the average and the marginal tax rate are increasing functions of income. With $c(y) := y - T(y) = ry^{1-\tau}$, the elasticity of after-tax income with respect to pretax income is constant and equal to $1-\tau$. Hence, τ is a measure of the progressivity of the tax code. Given τ, the parameter r governs how redistributive the tax system is: the larger r is, the more people receive transfers. Assuming logarithmic consumption utility, we can proceed along the same lines as for linear income taxation to show that y^* is a function of τ and ω, and that r is determined as a function of τ through the government budget constraint by $r(\tau) = \frac{E[y^*(\tau,\omega)]}{E[y^*(\tau,\omega)^{1-\tau}]}$. Policy preferences are now captured by

$$u(\tau,\omega) = \ln r(\tau) + (1-\tau)\ln y^*(\tau,\omega) - k(y^*(\tau,\omega),\omega). \tag{12}$$

[21] More formally, note that, by the envelope theorem, $u_1(\tau,\omega) = r'(\tau) - y^*(\tau,\omega)$. Using that y^* is a nondecreasing function of ω, we have $u_{12}(\tau,\omega) = -y_2^*(\tau,\omega) \leq 0$.

[22] It also implies that linear income taxation is covered by the equilibrium existence result in Proposition 5 below which provides conditions for the existence of a pure strategy equilibrium. For some of our results, we assume, moreover, that policy preferences are concave: $u_{11}(\tau,\omega) < 0$, for all τ and ω. In the linear income tax model, concavity holds, for instance, with an isoelastic effort cost function for $e \leq \frac{1}{2}$.

These preferences also satisfy the single-crossing property;[23] thus, a poorer agent would opt for a higher value of τ.

Again, Proposition 2 can be used to obtain a characterization of the political equilibrium tax system. Assuming that $\ln\omega$ is normally distributed with mean μ_ω and variance σ_ω^2, we obtain

$$(13) \qquad \frac{\tau^*}{1-\tau^*} = \left(1+\frac{1}{e}\right)\left((1-\tau^*)\,\sigma_\omega^2 - \mathrm{Cov}\left(\frac{\gamma^*(\omega)}{E[\gamma^*]}, \ln\omega\right)\right),$$

where the weights $\gamma^*(\cdot)$ are again given by (8). The left-hand side of this equation is increasing in τ^*; the right-hand side is decreasing in τ^*. The point of intersection is the uniquely determined equilibrium value of τ. As in the model of linear income taxation, high political weights on "the poor" yield high values of τ^*, and high political weights on "the rich" yield low values of τ^*.

Nonlinear Income Taxation.—The Mirrleesian approach to optimal income taxation does not impose any a priori restriction on the set of tax schedules. Here, this means that parties are not constrained by predetermined functional forms, but free to propose any tax schedule that satisfies the government budget constraint. They might, for instance, choose high marginal tax rates on "the rich" and earnings subsidies, i.e., negative marginal tax rates, for "the poor." Linear income taxes or *CRP* schedules are less flexible. Preferences are often assumed quasi-linear in consumption with isoelastic effort costs, see for instance Diamond (1998). Under these assumptions, online Appendix B.3 contains a derivation of policy preferences over nonlinear taxes. Using arguments from mechanism design, we show that a nonlinear income tax schedule can equivalently be represented by a bounded and monotonic earnings function $\mathbf{y}:\omega \mapsto \mathbf{y}(\omega)$. By the taxation principle (see Hammond 1979 or Guesnerie 1995) any tax schedule implements such an earnings function, and conversely, for any such earnings function one can find a tax schedule that implements it. Policy preferences can thus be represented by preferences over earnings functions, $u(\mathbf{y},\omega)$.[24]

The political equilibrium tax function $T^*:y \mapsto T^*(y)$ satisfies

$$(14) \qquad \frac{T^{*'}(\mathbf{y}^*(\omega))}{1-T^{*'}(\mathbf{y}^*(\omega))} = \left(1+\frac{1}{e}\right)\frac{1-F_\omega(\omega)}{\omega f_\omega(\omega)}\left(1-\Gamma^*(\omega)\right),$$

where $\mathbf{y}^*(\omega)$ is the equilibrium earnings of type ω and $T^{*'}(\mathbf{y}^*(\omega)$ is the corresponding marginal income tax rate. Moreover,

$$\Gamma^*(\omega) = E\left[\frac{\gamma^*(z)}{E[\gamma^*(\omega)]} \mid z \geq \omega\right],$$

[23] By the envelope theorem, $u_1(\tau,\omega) = \frac{r'(\tau)}{r(\tau)} - \ln y^*(\tau,\omega)$ and this implies $u_{12}(\tau,\omega) = -\frac{y_2^*(\tau,\omega)}{y^*(\tau,\omega)} \leq 0$, where the last inequality follows from $y_2^*(\tau,\omega) \geq 0$. Additional assumptions can be imposed to ensure concave policy preferences. Concavity holds, for instance, with isoelastic effort costs.

[24] In Section III we provide conditions that ensure the existence of pure and mixed strategy equilibria for this policy space.

where $\gamma^*(\cdot)$ are again given by (8), is the average political weight among people who are richer than type ω.

Equation (14) determines marginal tax rates in a political equilibrium with endogenous turnout.[25] The left hand side of this equation is an increasing function of the marginal income tax rate faced by agents with productive ability ω. Hence, the larger the right-hand side, the larger the marginal tax rate for these types in equilibrium. The right-hand side, in turn, is inversely related to the behavioral responses to taxation as measured by the elasticity parameter e. It is also inversely related to the hazard rate of the type distribution $\left(\omega f_\omega(\omega)\right)/\left(1 - F_\omega(\omega)\right)$. The logic is that a local increase in marginal taxes to be paid by types close to ω generates additional revenue from all individuals with a type above ω, i.e., from a mass of taxpayers equal to $1 - F_\omega(\omega)$. It provokes a behavioral response from all individuals with a type close to ω whose incentives to generate income are reduced. The size of the corresponding revenue loss is measured by $\omega f_\omega(\omega)$. Thus, the lower the hazard rate, the more revenue potential there is. Finally, the political weights determine the electoral return from exhausting this revenue potential. These returns are large if the average weight $\Gamma^*(\omega)$ among those who would have to pay this bill is small. By contrast, if people with types above ω have, on average, a lot of political weight, the marginal tax rate for type ω is low.

How Redistributive Is the Tax System?—Below, we turn to a comparative statics analysis that relates the level of taxes to the primitives of the model. As a preliminary step, we clarify how the political equilibrium weights characterized in Proposition 2 can be used to measure how redistributive a tax system is in a political equilibrium. Consider two specifications of the model's primitives giving rise to two different weighting functions that are respectively denoted by $\gamma_0^*:\omega \mapsto \gamma_0^*(\omega)$ and $\gamma_1^*:\omega \mapsto \gamma_1^*(\omega)$. Moreover, suppose that there is a decreasing function $\delta:\omega \mapsto \delta(\omega)$ with $E\left[\delta(\omega)\right] = 0$ so that

$$(15) \qquad \frac{\gamma_1^*(\omega)}{E\left[\gamma_1^*\right]} = \frac{\gamma_0^*(\omega)}{E\left[\gamma_0^*\right]} + \delta(\omega).$$

Thus, the weighting function γ_1^* assigns more weight to low income types and less weight to high income types. This implies that the equilibrium tax rate is higher with the weighting function γ_1^* in the model of affine income taxation. For the class of tax functions with a constant rate of progressivity, the equilibrium degree of progressivity is higher with weighting function γ_1^*. Finally, for fully flexible nonlinear taxes, marginal tax rates for all levels of income are higher under γ_1^*. Consequently, when condition (15) holds, we can order tax systems according to how redistributive they are. This order is robust in the sense that it does not depend on the fine details that distinguish different models of redistributive taxation.

[25] This formula is akin to the *ABC* formula for optimal, welfare-maximizing taxes due to Diamond (1998), except that the welfare weights in Diamond's formula are replaced by the political equilibrium weights derived in Proposition 2.

B. *Comparative Statics of Taxes and Turnout*

How does the level of redistributive taxation respond to changes in economic inequality? Why is turnout typically lower among "the poor" than among "the rich"? The first is a classic question in the political economy of taxation. A well-known hypothesis, due to Meltzer and Richard (1981), is that increases in inequality should lead to higher taxes. Moreover, an explanation for why taxes are not as high as predicted by such a median voter model is that the "decisive voter"[26] has an above-median income as "the poor" turn out in lower proportions than "the rich"; see Bénabou (2000). In this section, we discuss the implications of endogenous turnout for these questions.

As discussed in Kasara and Suryanarayan (2015), empirically the relationship between economic inequality, participation in the political process, and redistributive taxation is multifaceted. Our analysis cannot do justice to all these aspects; it can, however, illuminate one important channel. Consider an increase in polarization. In particular, suppose that the opponents of redistributive taxation become more zealous about a victory of their pro-market party, for reasons unrelated to taxation, such as, e.g., cultural issues, foreign policy, or other party characteristics that are orthogonal to questions of distribution. Then, our analysis implies that turnout becomes larger among "the rich" and that the equilibrium tax system becomes more redistributive. Thus, while political outcomes get tilted towards "the rich," tax policy gets tilted towards "the poor."

Symmetric Benchmark: We start by presenting a benchmark where turnout, although endogenous, does not vary with income, and determine the implications for taxes in equilibrium. Suppose that $\mathbf{B}^{1s} = \mathbf{B}^{2s}$ and $W^{1s} = W^{2s} \equiv W^s$, meaning that the parties have equal numbers of potential voters, and moreover, that the intensity of party preferences among the potential voters of party 1 is, on average, equal to the intensity of party preferences among the potential voters of party 2. We show in the online Appendix that, in equilibrium,

$$\frac{\sigma^{1*}}{\sigma^{2*}} = \left(\frac{W^{1s}/\mathbf{B}^{1s}}{W^{2s}/\mathbf{B}^{2s}}\right)^{\lambda} = 1. \tag{16}$$

These assumptions also determine the equilibrium value of the political weights. We have

$$\gamma_S^*(\omega) = \frac{1}{W^{1s}} B^s(\omega) + \frac{1}{W^{2s}} (1 - B^s(\omega)) = \frac{1}{W^s},$$

so that $\gamma_S^*(\omega)$ is now constant across types. Thus, for λ close to 1, the equilibrium tax policy maximizes an unweighted utilitarian welfare function.[27] Importantly, this finding does not depend on the within-base income distributions; e.g., utilitarianism prevails either if "the rich" overwhelmingly support party 1 and "the poor"

[26] Throughout, we define "the decisive voter" as the one whose ideal tax policy is implemented in equilibrium; see footnote 6.

[27] This observation extends to all possible values of λ if $\bar{q}(\omega)\, b^s(\omega)$ is also constant across types, implying that the inclination to swing from being a potential voter of party 1 to being a potential voter of party 2 does not depend on income.

overwhelmingly support party 2, or if the distribution across the parties' bases is even—all that matters is that the bases are of equal size.[28]

Taxes and Asymmetric Demobilization: In the following, we describe how shifts in the distribution of party preferences lead to departures from this symmetric benchmark for taxes and turnout. For ease of exposition, we focus on the case $\lambda = 1$ or, equivalently, on the case of linear voting costs. This enables us to highlight the implications of endogenous turnout for equilibrium taxes in the starkest possible way. We also impose the assumption that, all else equal, party 1 gets weakly more support from high-income voters and party 2 gets more support from low-income voters. We refer to party 1 as right leaning and to party 2 as left leaning. Formally, we assume that $B^s{:}\omega \mapsto B^s(\omega)$ is a nondecreasing function. Under these assumptions, higher turnout among "the rich" is equivalent to the turnout ratio σ^{1*}/σ^{2*} taking a value above one. We also assume that $W^{1s} \geq W^{2s}$, so that, in equilibrium, the stakes of the supporters of the right-leaning party are at least as large as the stakes of the supporters of the left-leaning party.

Consider a shift in idiosyncratic party preferences such that the ratio W^{1s}/W^{2s} increases. Recall that, as discussed in Section IB, W^{1s}/W^{2s} measures the intensity of political preferences of the supporters of party 1, relative to those of party 2. Thus, on average, the supporters of party 1 now feel more strongly about their party than do the supporters of party 2. Also assume that this change in preferences does not affect the size of the parties' bases $\mathbf{B}^{1s}$ and $\mathbf{B}^{2s}$, nor the within-base distributions by $B^{1s}{:}\omega \mapsto B^{1s}(\omega)$. For instance, such a shift takes place if mass is shifted from values of ε that are negative but close to zero to values that are much smaller; see Figure 1. Via equation (16) this implies that $\sigma^{1*}/\sigma^{2*} \geq 1$, so that turnout among "the rich" is not lower than turnout among "the poor," which, as discussed in the literature section, is the empirically plausible case.

PROPOSITION 3 (Taxes and Asymmetric Demobilization). *Suppose that* $W^{1s}/W^{2s} \geq 1$ *increases*, *keeping* B^{1s} *and* $B^{2s}{:}\omega \mapsto B^{2s}(\omega)$ *fixed. Then the tax system becomes more redistributive: the new equilibrium weighting function* γ_1^* *and the old weighting function* γ_0^* *satisfy* (15). *Moreover*, *the relative turnout ratio* σ^{1*}/σ^{2*} *and party* 1*'s winning probability go up.*

The results of Proposition 3 follow from party 1's trade-off between the returns from mobilizing its own base and the returns from demobilizing its competitor's base. These depend on the party's position in the electoral race: if its own supporters have, on average, stronger party biases than the opposition ($W^{1s} \geq W^{2s}$), and therefore have more at stake in the election, then by Proposition 1 party 1 has an edge in the race. By Proposition 2, this goes together with increased returns from demobilization. Since party 1 is the pro-market party and party 2 is the more interventionist

[28] Using a framework with CRP schedules, Heathcote, Storesletten, and Violante (2020) argue that US tax policy has been close to maximizing an unweighted utilitarian welfare function between the early 1980s and the early 2010s. For this period, Bierbrauer, Boyer, and Peichl (2021) document six reforms of the US federals income tax, three of which involved higher taxes on "the rich" and were enacted by Democratic governments, the other three were enacted by Republican governments and involved lower taxes on "the rich." This observation lends some plausibility to the balancedness condition that is needed for utilitarianism to prevail.

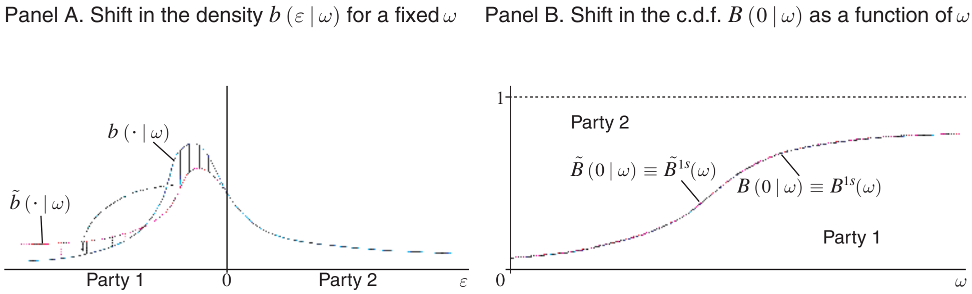

Figure 1. Comparative Statics

party, this leads to a *more* redistributive tax policy in equilibrium. Thus, somewhat paradoxically, the supporters of the pro-market party are not rewarded for their enthusiasm by a more market-oriented policy. They are rewarded by a victory of the party that has the label "pro-market," and the price to be paid for this victory is a less market-oriented policy. The driving force of this argument is the logic of *asymmetric demobilization*: by advocating more redistributive policies, party 1 depresses the turnout of its opponents, whose attachment to their party is, in comparison, weak, disproportionately more than it reduces the turnout of its own supporters.[29] As a result, the *decisive voter* becomes poorer at the same time that the turnout of "the rich" rises relative to that of "the poor."

"The rich" still benefit from all these developments. An application of the envelope theorem shows that the effect of an increase of W^{1s} on the payoff realized by the supporters of party 1 equals party 1's equilibrium probability of winning. Hence, it is positive.[30] When idiosyncratic party preferences for party 1 get stronger, this means that some dimension different from redistribution has become more urgent for its supporters. For instance, they might have become culturally more conservative and increasingly care about their party's position on gun rights, abortion or migration. This is also the dimension in which they are rewarded when their party wins. Concessions are made in redistributive taxation which is no longer as important as it has been prior to the preference shift.

The previous thought experiment consisted of changing the ratio W^{1s}/W^{2s} while keeping the size of the parties' bases and the within-base distributions $B^2(\cdot)$ fixed. fixed. The online Appendix contains further comparative statics results that reinforce the asymmetric demobilization logic. One of these results focuses on an increase in the size of the pro-market party's base, while keeping the within-base distributions xed (online Appendix B.4.3). Another keeps the size of the parties bases xed, but makes the pro-market party even more pro-market, i.e., its within-base distribution shifts so that its share of rich supporters increases further relative to its share of poor

[29] These observations are complemented by Proposition 6 in the online Appendix, which sheds light on what happens off equilibrium if the leading, pro-market, party does *not* take recourse to an asymmetric demobilization strategy.

[30] To see this, consider the payoff function in (4) and evaluate it in equilibrium. With equilibrium behavior of both parties and voters characterized by first-order conditions, the marginal effect of an increase of W^{1s} is simply the direct effect.

supporters (online Appendix B.4.4) In response to all of these changes, the equilibrium tax policy becomes more redistributive.[31]

All these results exploit the additive separability of policy preferences and party preferences. They imply that the fixed party characteristics do not matter for how voters are affected by the redistributive policies that the parties propose. If, by contrast, party 1 was less (more) competent in the design of redistribution programs than party 2, then the supporters of party 2 would be less (more) impressed if party 1 proposed high taxes and the asymmetric demobilization strategy would be less (more) effective. Thus, additive separability is a benchmark, and the force of asymmetric demobilization may become stronger or weaker with additional considerations such as competence, valence, or an interdependence of party platforms and fixed party positions.

C. *Taxes*, *Turnout*, *and Inequality: A Parametric Example*

We now explore the implications of our framework for the classic question of whether increases in economic inequality lead, via the political process, to more redistributive taxation.[32] Throughout, we assume that productivity types ω are log-normally distributed with parameters $\left(\mu_\omega, \sigma_\omega^2\right)$, and we use the variance σ_ω^2 as the measure of economic inequality.

To be specific, we employ the model of affine income taxation. We apply a monotone and concave transformation $U{:}u(\tau,\omega) \mapsto U\big(u(\tau,\omega)\big)$ to the policy preferences described in equation (11): thus, a type ω-individual votes for party 1 if $U\big(u(\tau^1,\omega)\big) - U\big(u(\tau^1,\omega)\big) \geq \varepsilon$. Doing so ensures that the symmetric benchmark tax rate is strictly positive, the empirically plausible case. The weighting function $\gamma^*(\omega)$ characterized in Proposition 2 is then replaced by $\gamma_U^*(\omega) = U'\Big(u\big(\tau^*,\omega\big)\Big)\,\gamma^*(\omega)$, and the formula (11) characterizing the equilibrium tax rate is otherwise unchanged. For specificity, we let $U(u) = \ln u$.

As our analysis of Section IIB has shown, with endogenous turnout, what economic inequality implies for redistribution depends on whether the political competition is balanced—in which case the equilibrium tax policy is utilitarian—or tilted in favor of one party. The following assumptions enable us to distinguish between these different political scenarios in a tractable way. First, we suppose that idiosyncratic party preferences are, for each type ω, uniformly distributed:

$$B\Big(u\big(p^1,\omega\big) - u\big(p^2,\omega\big) \mid \omega\Big) = \alpha(\omega) + \beta(\omega)\Big[u\big(p^1,\omega\big) - u\big(p^2,\omega\big)\Big],$$

[31] These comparative statics results can also be used to think through the implications of systematic changes in party allegiances. For the United States, Enke, Polborn, and Wu (2021, p. 1) document the following pattern: "*rich-but-socially-liberal voters* (*the educational elite*) *switched from Republicans to Democrats*, *while poor-but-socially-conservative voters* (*e.g.*, *manufacturing workers*) *switched from Democrats to Republicans*." In our model, this can be represented by a fraction of the rich supporters of party 1 switching to the base of party 2, and a fraction of the poor supporters of party 2 switching to the base of party 1. Our comparative statics results in the online Appendix suggest that such a preference shift leads parties to propose less redistributive tax policies in equilibrium.

[32] The hypothesis that more inequality yields higher taxes is due to Meltzer and Richard (1981). Bénabou (2000), by contrast, derives a U-shaped relation between economic inequality and the level of taxation.

so that $B^s(\omega) = \alpha(\omega)$. Second, we assume that $\alpha(\omega) \in (0,1)$ is increasing and given by the c.d.f. of a log-normal distribution with parameters $(\mu_\alpha, \sigma_\alpha^2)$,

$$\alpha(\omega) = \frac{1}{\sigma_\alpha\sqrt{2\pi}}\int_0^\omega \frac{1}{w}\exp\left(-\frac{(\log w - \mu_\alpha)^2}{2\sigma_\alpha^2}\right)dw.$$

This functional form allows us to disentangle two different sources of political inequality: differences in the size of the parties' bases, on the one hand, and differences in the within-base income distributions, on the other. To see this, Figure 2 depicts the function $\alpha{:}\omega \mapsto \alpha(\omega)$ as the mean parameter μ_α varies (panel A), and as the variance parameter σ_α^2 varies (panel B). Lower values of μ_α shift up the support for party 1 at every income level, and hence raise its aggregate base B^{1s}, without affecting the variance of this support. On the other hand, σ_α^2 determines how polarized the electorate is for fixed aggregate bases: a low value of σ_α^2 implies that "the rich" overwhelmingly support party 1 while "the poor" overwhelmingly support party 2; a high value σ_α^2 implies that all income groups are split between the two groups of supporters in the same proportion as the whole population.

Panel A of Figure 3 shows the relationship between economic inequality and equilibrium tax rates for different sizes of party 1's base, keeping polarization σ_α constant. The asymmetric demobilization logic is visible here: the stronger the pro-market party, the larger is the equilibrium tax rate. An increase in economic inequality σ_ω^2 makes the covariance between weights $\gamma_U^*(\omega)$ and incomes more negative, and this pushes the equilibrium tax rate up. This holds irrespective of which party is dominant. Panel B of Figure 3 keeps economic inequality $(\sigma_\omega^2 = 0.5)$ constant, and shows the relationship between polarization σ_α^2 and equilibrium tax rates for different sizes of party 1's base. Polarization leads to more extreme equilibrium taxes—either above or below utilitarianism depending on whether the pro-market party is strong or weak.

The interaction between economic and political inequality can dampen, and potentially reverse, the increase in equilibrium taxes that results from an increase in economic inequality. Suppose that some middle-class people get poorer and some get richer, while everyone keeps their party preferences. Thus, economic inequality goes up while political inequality goes down. Given an initial situation in which $B^s{:}\omega \mapsto B^s(\omega)$ is increasing, among the new rich are more people who support the left-leaning party, and among the new poor are more people who support the pro-market party. If the initial situation is one with a strong pro-market party, the previous analysis implies that this decrease in polarization pushes equilibrium taxes down, and hence counteracts the increase in taxes due to the higher level of economic inequality.

To explore the quantitative importance of this effect in the parametric example of this section, suppose that economic inequality, measured by the variance of log-wages, increases from σ_ω^2 to $\tilde{\sigma}_\omega^2$, while their mean μ_ω remains fixed. Log-wages after the rise in inequality are represented by the random variable $\log\tilde{\omega} \sim \mathcal{N}(\mu_\omega, \tilde{\sigma}_\omega^2)$. Since we suppose that political preferences are attached to people (as opposed to productivity types, as in panel A of Figure 3), we must construct the path of each individual agent that generates this shift in the wage distribution.

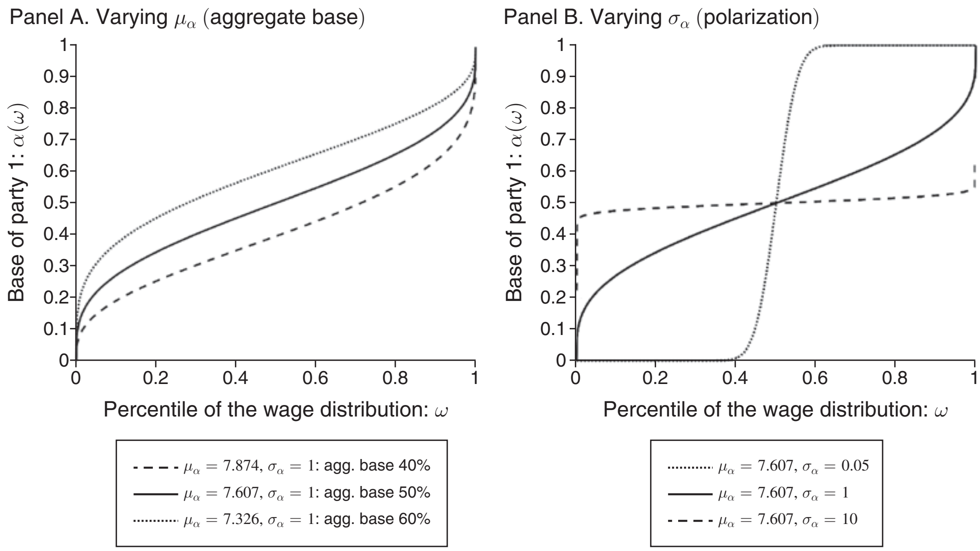

Figure 2. Base of Party 1 along the Income Distribution

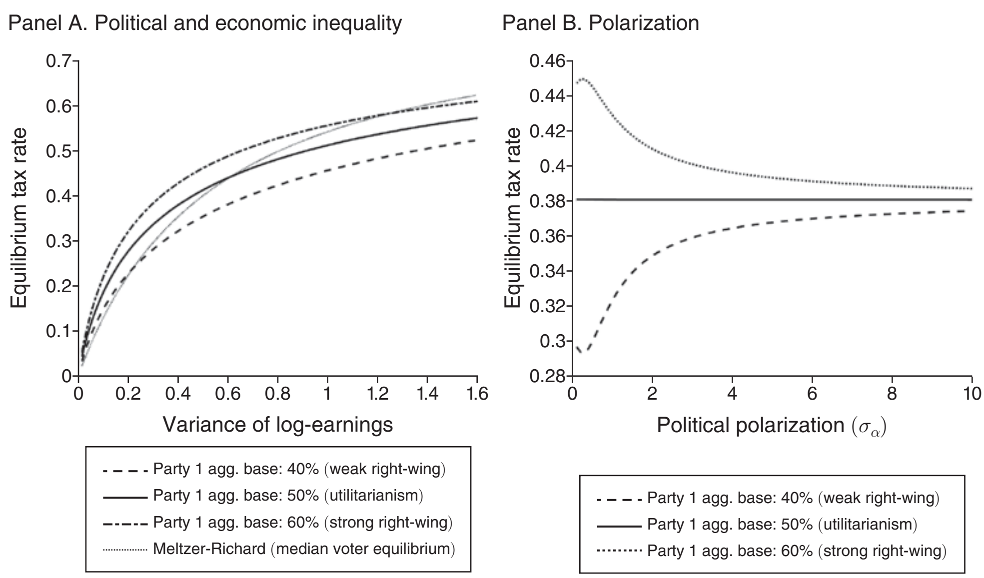

Figure 3. Comparative Statics

A natural way to do so is to note that we can write $\log\tilde{\omega} = \log\omega + X$, for some random variable $X \sim \mathcal{N}(0,\sigma_X^2)$ independent of ω, with $\sigma_X^2 = \tilde{\sigma}_\omega^2 - \sigma_\omega^2$; the distribution $\varphi_{\log\tilde{\omega}}$ of $\log\tilde{\omega}$ is then given by the convolution $\varphi_{\log\omega} * \varphi_X$ of the distributions of $\log\omega$ and X. An interpretation of this convolution is that agents with log-wage $\log\omega$ are spread out according to a Gaussian distribution centered in $\log\omega$ and with variance σ_X^2. That is, for any $x \in \mathbb{R}$, agents with initial log-wage $\log\omega$ draw the new log-wage level $\log\tilde{\omega} = \log\omega + x$ with probability $\varphi_X(x)$.[33] Consistent with this interpretation, we suppose that all agents with initial log-wage $\log\omega$ keep their original political preferences ε after the rise in inequality, regardless of their realized productivity shock x (and corresponding new log-wage level $\log\tilde{\omega}$). The aggregate mass of supporters of party 1 after the perturbation is then given by

$$\begin{aligned}\tilde{\mathbf{B}}^{1s} &= \int_{-\infty}^{\infty}\Big[\int_{-\infty}^{\infty}\alpha\big(e^{\log\omega}\big)\,\varphi_{\log\omega}(\log\omega)\,\varphi_X(\log\tilde{\omega}-\log\omega)\mathrm{d}\log\omega\Big]\mathrm{d}\log\tilde{\omega}\\ &= \int_{-\infty}^{\infty}\alpha\big(e^{\log\omega}\big)\,\varphi_{\log\omega}(\log\omega)\mathrm{d}\log\omega \;=\; \mathbf{B}^{1s},\end{aligned}$$

and the aggregate intensity of preferences among party 1's supporters is

$$\tilde{W}^{1s} = \int_{\mathbb{R}^2}\int_{\mathbb{R}_-}|\varepsilon|dB\big(\varepsilon\,|\,e^{\log\omega}\big)\,\varphi_{\log\omega}(\log\omega)\,\varphi_X(\log\tilde{\omega}-\log\omega)\mathrm{d}\log\omega\mathrm{d}\log\tilde{\omega} = W^{1s};$$

that is, the rise in inequality, and the corresponding shift in the distribution of political preferences, affect neither the size of the parties' base, nor the intensity of preferences of their sets of supporters.[34] Hence, our comparative statics experiment isolates the pure effect of the reduced polarization of political preferences along the wage distribution.

The quantitative results are shown in Figure 4. In panel A, we plot the change in the political weights $\gamma(\omega)/E\gamma$ following the rise in inequality, that is, the function $\delta(\omega)$ defined in equation (15). We compute this change in equilibrium weights following an increase in the variance of log-earnings from 0.07 to 0.12 (solid line), 0.53 (dotted line), and 1.43 (dashed line). As we anticipated, the equilibrium resulting from a rise in inequality assigns lower weights to the poor, and higher weights to the rich, relative to the initial equilibrium.[35,36] Ceteris paribus, this implies that the equilibrium tax system will tend to be less redistributive after the rise in inequality. This insight concerns the characterization of the equilibrium political weights, and therefore applies whether taxes are affine, *CRP*, Mirrleesian, etc.

Panel B of Figure 4 focuses on the case of affine tax schedules, and plots the equilibrium tax rate as a function of the variance of log earnings. The dotted line depicts the tax rate chosen by the median voter, as in Meltzer and Richard (1981). The dashed line represents the tax rate in our model of endogenous turnout when the aggregate base of party 1 is 60 percent,[37] assuming that agents adopt the political

[33] To see that this transformation indeed leads to a Gaussian distribution $\mathcal{N}(\mu_\omega,\tilde{\sigma}_\omega^2)$, note that the mass of agents at a given log-wage level $\log\tilde{\omega}$ after the rise in inequality can be expressed as $\int_{-\infty}^{\infty}\varphi_{\log\omega}(\log\tilde{\omega}-x)\,\varphi_X(x)dx = (\varphi_{\log\omega}*\varphi_X)(\log\tilde{\omega}) \sim \mathcal{N}(\mu_\omega,\tilde{\sigma}_\omega^2)$.

[34] The political weights $\gamma(\omega)$ defined in (8) do not remain fixed, however, since the base of party 1 conditional on each particular wage group, $B^s(\omega)$, changes.

[35] To interpret the scale on the y-axis, recall that the political weights $\gamma(\omega)/E\gamma$ are, by construction, equal to one on average.

[36] The x-axis gives log-wages $\log\omega$ rather than log-earnings $\log y$, since earnings are endogenous to the tax rate. In our setting with linear taxes, we have $\log y(\omega) = (1+e)\log\omega + e\log(1-\tau)$, and our calibration assumes a labor supply elasticity $e = 0.33$.

[37] Our calibration assumes a uniform swing $\beta(\omega) = 0.5$ percent for all ω. We obtain a ratio of intensities of political preferences equal to $W^{1s}/W^{2s} = 2.2$. Together with $\mathbf{B}^{1s} = 0.6$, this leads to a ratio of turnout rates $\sigma^{1*}/\sigma^{2*} = 1.47$. A value $\sigma^{1*}/\sigma^{2*} > 1$ is the empirically relevant case as it implies that turnout is biased in favor of the rich.

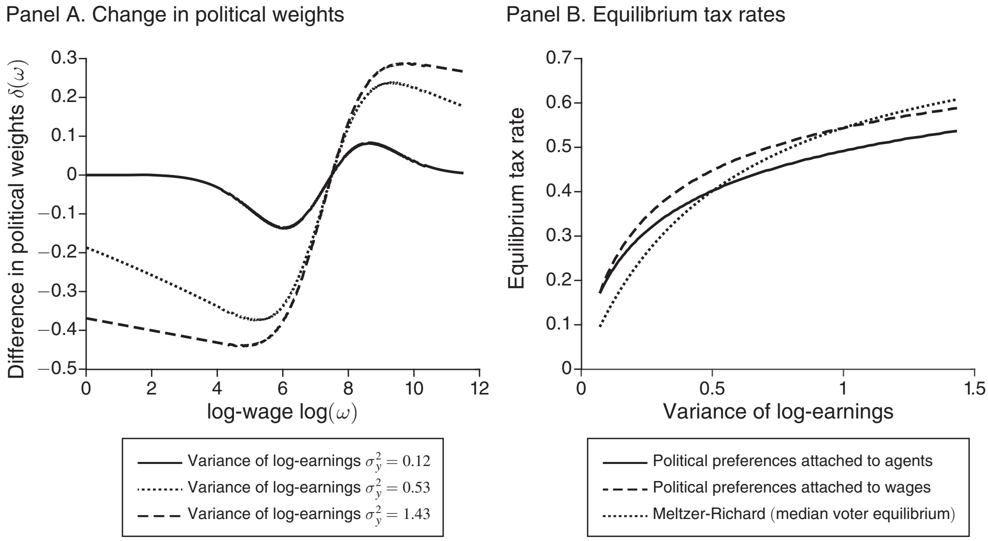

Figure 4. Comparative Statics: Inequality and Redistribution

preferences associated with their new wage group when their productivity changes. These two lines are the same as those in Panel A of Figure 3. By contrast, the solid line is obtained by assuming that agents keep their former political preferences as their wages change. Consistent with the change in the shape of political weights depicted in panel A, the uniformization of political preferences across wages that results from the rise in inequality weakens the equilibrium relationship between inequality and redistribution. In this particular parametric example, this opposing force is quantitatively too small to offset the decay in marginal utilities, and hence does not reverse the relationship between inequality and taxes.

More generally, however, we could construct a rise in inequality and choose the model's primitives such that the decrease in political polarization becomes the dominant force. For instance, with quasi-linear preferences and linear taxation, if the initial situation has $W^{1s}/W^{2s} > 1$ and the new situation has $W^{1s}/W^{2s} = 1$, then, by Propositions 2 and 3, taxes are initially positive, and drop to zero after the increase in economic inequality. In this scenario, the rise in inequality is accompanied by a decrease in the relative aggregate intensity of preferences of the supporters of the "right-wing" party (i.e., the party that is disproportionately supported by rich voters), which in turn reverses the sign of the relationship between economic inequality and redistribution.

III. Existence of Equilibrium

An Example: We begin by imposing specific assumptions on the voting cost function and the distribution of idiosyncratic party preferences. Under these conditions the existence of a pure strategy equilibrium is ensured with linear income taxes, *CRP* schedules and, most notably, nonlinear income taxes. Subsequently, we provide conditions for equilibrium existence that transcend this special case.

ASSUMPTION 2: *Suppose that voting costs are linear so that* $\lambda = 1$. *Also suppose that idiosyncratic party biases are, for each type* ω, *uniformly distributed:*

$$B\big(u(p^1,\omega) - u(p^2,\omega) \mid \omega\big) = \alpha(\omega) + \beta(\omega)\big[u(p^1,\omega) - u(p^2,\omega)\big].$$

Moreover, the distributions have a wide support and are close to symmetric. Formally,

(*i*) *There exists* $\bar{\beta}$ *close to zero so that* $0 < \beta(\omega) \leq \bar{\beta}$, *for all* ω.

(*ii*) *There exists* $\bar{\alpha}$ *close to zero so that,* $\alpha(\omega) \in \left[\frac{1}{2} - \bar{\alpha}, \frac{1}{2} + \bar{\alpha}\right]$.

Under Assumption 2, with a one-dimensional policy space, $\mathcal{P} = [\underline{p}, \bar{p}] \subset \mathbb{R}$, and concave policy preferences $u(\cdot\,,\omega)$ for each type ω, it is straightforward to verify that $\Pi^1(\cdot\,,p^2){:}p^1 \mapsto \Pi^1(p^1,p^2)$ is a globally concave function for every value of p^2, and that $\Pi^1(p^1,\cdot){:}p^2 \mapsto \Pi^1(p^1,p^2)$ is globally convex for every value of p^1. Consequently, with concave policy preferences, the regularity conditions stated in Section IB hold for linear income taxes and *CRP* schedules, ensuring the existence of a unique pure strategy equilibrium, that is, moreover, symmetric (see Proposition 2). The following Proposition extends this observation to fully nonlinear income taxes.

PROPOSITION 4: *Suppose that Assumption 2 holds. Let* $\mathcal{P}$ *be the space of nonnegative, bounded and monotonic earnings functions. Suppose that the regularity conditions in online Appendix D.1 are satisfied.*[38] *Also suppose that preferences are quasi-linear in consumption and that effort costs are isoelastic. Then there is a unique pure strategy equilibrium. This equilibrium is symmetric.*

The proof involves the following steps. We first use functional derivatives to derive first-order conditions that characterize the parties' best responses. This gives us a candidate for a symmetric equilibrium in pure strategies. We then show that this equilibrium candidate also satisfies the second-order conditions. Thus, parties have no incentive to deviate locally. Finally, we invoke the contraction mapping theorem to show that, under Assumption 2, there is one and only one policy that is a best response to itself and that this policy coincides with the equilibrium candidate. Hence, there neither is an incentive to deviate to a policy that is not in a neighborhood of the equilibrium candidate.

For probabilistic voting models with exogenous turnout, it is known that pure strategy equilibria exist under regularity conditions on the distributions of idiosyncratic party preferences.[39] In particular, it is known that, with i.i.d. distributions of idiosyncratic party biases, probabilistic voting gives rise to an equilibrium outcome that maximizes a weighted utilitarian welfare function; see Banks and Duggan (2005). If

[38] These regularity conditions are familiar from the literature on optimal welfare-maximizing taxation. They ensure that it suffices to study a relaxed best response problem that does not explicitly impose a monotonicity constraint on earnings functions. If the regularity conditions are met, the solution to the relaxed problem is monotonic.

[39] See, for instance, the seminal paper by Lindbeck and Weibull (1987).

those distributions are, moreover, uniform, there is a dominant strategy equilibrium in which the parties maximize an unweighted utilitarian welfare function.[40]

Proposition 4 shows that these findings extend to a framework with endogenous turnout. Our analysis, moreover, moves beyond the special case of i.i.d. and uniform distributions by means of a continuity argument: existence and uniqueness of equilibrium also holds for distributions that are sufficiently "close" to that benchmark. The assumption that the uniform distributions have a wide support implies that strategic substitutes and complements play a limited role. In the limit, i.e., for $\bar{\beta} = 0$, equilibria are in dominant strategies, and best responses no longer depend on the tax policy proposed by the other party. The assumption that the distributions are close to symmetric implies that the race between the two parties is close. Thus, in a neighborhood of such a symmetric, dominant strategy equilibrium we can be assured that an equilibrium exists, and is unique.

Affine and CRP Tax Schedules: Pure Strategies: With a one-dimensional policy space, a pure-strategy equilibrium can be shown to exist under conditions that are weaker than those invoked in Proposition 4.

PROPOSITION 5: *Let $\mathcal{P}$ be a compact subset of $\mathbb{R}$. Suppose that $u:(p,\omega) \mapsto u(p,\omega)$ is, for all ω, a continuously differentiable function of p, and that every type ω has an ideal policy in the interior of $\mathcal{P}$. Assume moreover that preferences satisfy the single-crossing property, $u_{12}(p,\omega) \leq 0$ for all $p \in \mathcal{P}$ and $\omega \in \Omega$.*

(*i*) *Suppose there are scalars a and $b > 0$, so that, for all $(p,p') \in \mathcal{P}^2$,*

$$\Pi^1(p,p') = a + b\,\Pi^2(p',p), \quad \textit{where} \quad \Pi^2(p',p) := 1 - \Pi^1(p',p). \tag{17}$$

Then there is a symmetric equilibrium in pure strategies.

(*ii*) *Alternatively, suppose that utility functions are concave in p. Also suppose that for every p^2, there is at most one p^1 so that*

$$\Pi^1_1(p^1,p^2) = 0 \quad \textit{and} \quad \Pi^1_{11}(p^1,p^2) < 0, \tag{18}$$

where Π^1_1 is the first and Π^1_{11} the second derivative of Π^1 with respect to p^1, and analogously for party 2. Then there is a unique equilibrium in pure strategies. This equilibrium is symmetric.

Proposition 5 provides different sufficient conditions for the existence of a symmetric pure-strategy equilibrium. Part (i) involves a condition of symmetry. To interpret condition (17), suppose first that $a = 0$ and $b = 1$. Then the condition becomes $\Pi^1(p,p') = \Pi^2(p',p)$; i.e., if the parties flip their policies, so do their winning prob-

[40] This finding can be squared with a Mirrleesian model of taxation because a party's best response problem now coincides with a utilitarian problem of welfare-maximization; see, e.g., Farhi et al. (2012) for an application of this insight.

abilities. This condition holds, for instance, if all the distributions B in our model are symmetric. Condition (17) is a generalization of this case of perfect symmetry, allowing both for a fixed advantage $(a < 0)$ or disadvantage $(a > 0)$ of party 2 relative to party 1, as well as for the possibility that a platform change that increases the winning probability of party 1 would have less $(b < 1)$ or more $(b > 1)$ of an impact on party 2's winning probability.

Part (i) provides a condition for existence that is parsimonious in the sense that it does not involve any assumption on the curvature of the functions $\{B(\cdot\,|\,\omega) : \omega \in \Omega\}$ that describe the party preferences of different types, nor on the functions $\{u(\cdot\,,\omega) : \omega \in \Omega\}$ that describe the policy preferences of different types. The proof consists in showing that the best response function of party 1 has a fixed point by Brouwer's fixed point theorem, and then to show that, under condition (17), any such fixed point is a saddle point of Π^1. No curvature assumption is needed along the way.

Part (ii) is parsimonious in a different way. It avoids any assumption of symmetry, but imposes an assumption on curvature. This assumption ensures that any policy that satisfies the first- and the second-order conditions of a best response problem is in fact the solution to this problem. Equipped with this regularity condition, we show that the best response functions of parties 1 and 2 have identical fixed points and, exploiting the concavity of u, that there is only one such fixed point. As a corollary we obtain the existence, uniqueness and symmetry of an equilibrium in pure strategies.

The content of the curvature assumption can be most easily demonstrated under the assumption of linear voting costs, or $\lambda = 1$, so that the objective of party 1 is to maximize $\Pi^1(p^1,p^2) = W^1(p^1,p^2)/W^2(p^1,p^2)$. As we show in the online Appendix, (18) holds if, for all $(p^1,p^2) \in \mathcal{P}^2$,

$$\frac{b(\Delta u(\cdot) \mid \omega)}{B(\Delta u(\cdot) \mid \omega)} \leq \left| \frac{u_{11}(p^1,\omega)}{u_1(p^1,\omega)} \right|, \tag{19}$$

whenever ω is such that $u_1(p^1,\omega) > 0$, and

$$\frac{b(\Delta u(\cdot)\,|\,\omega)}{1 - B(\Delta u(\cdot)\,|\,\omega)} \leq \left| \frac{u_{11}(p^1,\omega)}{u_1(p^1,\omega)} \right|, \tag{20}$$

whenever ω is such that $u_1(p^1,\omega) < 0$. We use $\Delta u(\cdot)$ as a shorthand for $u(p^1,\omega) - u(p^2,\omega)$. To interpret these conditions, consider a marginal increase in the policy p^1. For individuals who benefit from such a change, condition (19) relates the percentage change in political support for party 1 to the percentage change in marginal utility. For individuals who are harmed by the policy shift, condition (20) relates the percentage change in political support for party 2 to the percentage change in marginal utility. Both conditions require that the percentage change in political support must not be larger than the percentage change in individual welfare. Broadly, the effect that is driven by idiosyncratic party preferences must not outweigh the effect that is driven by policy preferences.

As a more specific example, suppose that idiosyncratic party biases are, for each type ω, uniformly distributed: $B(\Delta u(\cdot)\,|\,\omega) = \alpha(\omega) + \beta(\omega)\Delta u(\cdot)$. The wider the support of the uniform distribution, the lower $\beta(\omega)$ is. Hence, if all distributions are "close to uniform over the reals," then the left-hand sides of both (19) and (20) are "close to zero," with the implication that these inequalities hold.

The assumption of single-crossing preferences is indispensable for the existence of a pure strategy equilibrium. As is well known, symmetric zero-sum games, such as "matching pennies," do not typically have equilibria in pure strategies. In our setting, the single-crossing property implies that equilibrium policies belong to an interior set of Pareto-efficient policies that coincides with the set of the different voter types' ideal polices.[41] This also implies that the equilibrium policy admits an interpretation as the ideal policy of a *decisive voter*.

Second, as an implication of the single-crossing property, the voters' policy preferences over the set of Pareto-efficient policies are single-peaked. Single-peakedness and single-crossing preferences are typically viewed as unrelated sufficient conditions that enable a proof of a median voter theorem in models without idiosyncratic party preferences. It is therefore interesting to note that, in our setting, the single-crossing property implies single-peakedness.

Nonlinear Income Taxation: Mixed Strategies: The case of fully nonlinear income tax schedules or, equivalently, of bounded and monotonic earnings functions, is a high-dimensional policy space. For such spaces, the existence of a pure-strategy equilibrium cannot generally be expected. We can, however, provide a generic existence proof for a mixed strategy equilibrium. *Suppose that $\mathcal{P}$ is the space of nonnegative, bounded and monotonic functions in a compact space. Suppose that $p_a \to p_b$, in the sense of uniform convergence, implies that $u(p_a, \omega) \to u(p_b, \omega)$ for all $\omega \in \Omega$. Then there exists an equilibrium in mixed strategies.*

Our proof is based on Glicksberg's existence theorem for mixed-strategy equilibria of zero-sum games. The application of Glicksberg's theorem is not straightforward. The previous literature contains existence proofs for mixed strategy equilibria based on a multidimensional Euclidian policy spaces (see, e.g., Banks and Duggan 2005) but not for the space of nonnegative, bounded and monotonic functions. In the proof we verify that—with an appropriate notion of convergence applied to the space of nonnegative, bounded and monotonic functions—the premises of Glicksberg's theorem hold.

As argued by Laslier (2000), mixed strategies need not literally be interpreted as randomization devices. During a campaign, politicians may talk differently to different audiences, and remain vague most of the time. For instance, they may say both "the hard-working middle-class people must not be left behind" and "we cannot afford to discourage the productive efforts of the most talented," etc. Presumably, they differ in emphasis or in how often they say one thing or the other depending on the audience they are addressing. Such a strategy can be interpreted as a mixed strategy.[42]

[41] In games with strategic complementarities, a single-crossing property implies the existence of pure strategy equilibria; see, e.g., Amir, Jakubczyk, and Knauff (2008) and the references therein. Here this does not apply. In the game between the two parties such complementarities do not arise.

[42] This also connects to a political science literature on the "blurring" of party positions; see Han (2020).

IV. Concluding Remarks

This paper presents an analysis of political competition with endogenous turnout. Two parties choose platforms to maximize their probability of winning an election and thereby take into account the implications of their platform choice for turnout. If a left-leaning party differentiates itself from a pro-market party and proposes an expansion of the welfare state or higher taxes on top incomes, this gives its supporters incentives to turn out to vote: after all, there is now a political alternative worth fighting for. However, the stakes are also higher for the supporters of the pro-market party. From their perspective, there is now an increased urgency to prevent the worst case, a victory of the socialists.

Our equilibrium analysis pins down the strengths of these forces when parties best reply to each other in an attempt to find the optimal trade-off between the turnout incentives for their own supporters and the supporters of the competing party. While both parties ultimately propose the same platform, there is an asymmetry in this trade-off when the race between the parties is unbalanced. Say that the party preferences are such that the pro-market party is the likely winner of the election. Then, in equilibrium, it assigns more weight to the demobilization of the left-leaning voters than to the mobilization of its own conservative base. The left-leaning party, by contrast, puts more weight on the mobilization of its own supporters. Thus, both parties arrive at the same conclusion, the equilibrium platform, but for different reasons.

REFERENCES

Acemoglu, Daron, Michael Golosov, and Aleh Tsyvinski. 2008. "Political Economy of Mechanisms." *Econometrica* 76 (3): 619–41.

Acemoglu, Daron, Mikhail Golosov, and Aleh Tsyvinski. 2010. "Dynamic Mirrlees Taxation under Political Economy Constraints." *Review of Economic Studies* 77 (3): 841–81.

Adams, James, and Samuel Merrill III. 2003. "Voter Turnout and Candidate Strategies in American Elections." *Journal of Politics* 65 (1): 161–89.

Aldashev, Gani. 2015. "Voter Turnout and Political Rents." *Journal of Public Economic Theory* 17 (4): 528–52.

Alesina, Alberto, and George-Marios Angeletos. 2005. "Fairness and Redistribution." *American Economic Review* 95 (4): 960–80.

Alger, Ingela, and Jean-François Laslier. 2021. "Homo Moralis Goes to the Voting Booth: A New Theory of Voter Turnout." Unpublished.

Amir, Rabah, Michał Jakubczyk, and Małgorzata Knauff. 2008. "Symmetric versus Asymmetric Equilibria in Symmetric Supermodular Games." *International Journal of Game Theory* 37 (3): 307–20.

Banks, Jeffrey S., and John Duggan. 2005. "Probabilistic Voting in the Spatial Model of Elections: The Theory of Office-Motivated Candidates." In *Social Choice and Strategic Decisions: Essays in Honor of Jeffrey S. Banks*, edited by David Austen-Smith and John Duggan, 15–56. Berlin: Springer.

Bénabou, Roland. 2000. "Unequal Societies: Income Distribution and the Social Contract." *American Economic Review* 90 (1): 96–129.

Bernhardt, Dan, Peter Buisseret, and Sinem Hidir. 2018. "The Race to the Base." Unpublished.

Bierbrauer, Felix J., and Pierre C. Boyer. 2016. "Efficiency, Welfare, and Political Competition." *Quarterly Journal of Economics* 131 (1): 461–518.

Bierbrauer, Felix J., Pierre C. Boyer, and Andreas Peichl. 2021. "Politically Feasible Reforms of Nonlinear Tax Systems." *American Economic Review* 111 (1): 153–91.

Bierbrauer, Felix, Aleh Tsyvinski, and Nicolas Werquin. 2022. "Replication Data for: Taxes and Turnout: When the Decisive Voter Stays at Home." American Economic Association [publisher], Inter-university Consortium for Political and Social Research [distributor]. https://doi.org/10.3886/E149781V1.

Brett, Craig, and John A. Weymark. 2017. "Voting over Selfishly Optimal Nonlinear Income Tax Schedules." *Games and Economic Behavior* 101: 172–88.

Callander, Steven, and Catherine H. Wilson. 2007. "Turnout, Polarization, and Duverger's Law." *Journal of Politics* 69 (4): 1047–56.

Chen, Jowei. 2013. "Voter Partisanship and the Effect of Distributive Spending on Political Participation." *American Journal of Political Science* 57 (1): 200–17.

Coate, Stephen, and Michael Conlin. 2004. "A Group Rule-Utilitarian Approach to Voter Turnout: Theory and Evidence." *American Economic Review* 94 (5): 1476–1504.

Coate, Stephen, Michael Conlin, and Andrea Moro. 2008. "The Performance of Pivotal-Voter Models in Small-Scale Elections: Evidence from Texas Liquor Referenda." *Journal of Public Economics* 92 (3–4): 582–96.

Coughlin, Peter, and Shmuel Nitzan. 1981. "Electoral Outcomes with Probabilistic Voting and Nash Social Welfare Maxima." *Journal of Public Economics* 15 (1): 113–21.

Cox, Gary W. 2010. "Swing Voters, Core Voters, and Distributive Politics." In *Political Representation,* edited by Ian Shapiro, Susan C. Stokes, Elisabeth Jean Wood, and Alexander S. Kirshner, 342–57. Cambridge: Cambridge University Press.

Degan, Arianna, and Antonio Merlo. 2011. "A Structural Model of Turnout and Voting in Multiple Elections." *Journal of the European Economic Association* 9 (2): 209–45.

Diamond, Peter A. 1998. "Optimal Income Taxation: An Example with a U-Shaped Pattern of Optimal Marginal Tax Rates." *American Economic Review* 88 (1): 83–95.

Enke, Benjamin, Mattias Polborn, and Alex A. Wu. 2021. "An Economic Theory of Moral and Political Change." Unpublished.

Farhi, Emmanuel, Christopher Sleet, Iván Werning, and Sevin Yeltekin. 2012. "Non-linear Capital Taxation without Commitment." *Review of Economic Studies* 79 (4): 1469–93.

Fahri, Emmanuel, and Iván Werning. 2008. "The Political Economy of Non-linear Capital Taxation." Unpublished.

Feddersen, Timothy J. 2004. "Rational Choice Theory and the Paradox of Not Voting." *Journal of Economic Perspectives* 18 (1): 99–112.

Feddersen, Timothy, and Alvaro Sandroni. 2006. "A Theory of Participation in Elections." *American Economic Review* 96 (4): 1271–82.

Forschungsgruppe Wahlen, Mannheim. 2013a. "Politbarometer Ost 2009 (Cumulated Data Set, incl. Flash)."

Forschungsgruppe Wahlen, Mannheim. 2013b. "Politbarometer West 2009 (Cumulated Data Set, incl. Flash)."

Forschungsgruppe Wahlen, Mannheim. 2015. "Politbarometer 2013 (Cumulated Data Set, incl. Flash)."

Forschungsgruppe Wahlen, Mannheim. 2018. "Politbarometer 2017 (Cumulated Data Set)."

Gans, Joshua, and Michael Smart. 1996. "Majority Voting with Single-Crossing Preferences." *Journal of Public Economics* 59 (2): 219–37.

Guesnerie, Roger. 1995. *A Contribution to the Pure Theory of Taxation.* Cambridge, UK: Cambridge University Press.

Hammond, Peter J. 1979. "Straightforward Individual Incentive Compatibility in Large Economies." *Review of Economic Studies* 46 (2): 263–82.

Han, Kyung Joon. 2020. "Beclouding Party Position as an Electoral Strategy: Voter Polarization, Issue Priority and Position Blurring." *British Journal of Political Science* 50 (2): 653–75.

Harsanyi, John C. 1980. "Rule Utilitarianism, Rights, Obligations and the Theory of Rational Behavior." *Theory and Decision* 12 (2): 115–33.

Heathcote, Jonathan, Kjetil Storesletten, and Giovanni L. Violante. 2017. "Optimal Tax Progressivity: An Analytical Framework." *Quarterly Journal of Economics* 132 (4): 1693–1754.

Heathcote, Jonathan, Kjetil Storesletten, and Giovanni L. Violante. 2020. "Presidential Address 2019: How Should Tax Progressivity Respond to Rising Income Inequality?" *Journal of the European Economic Association* 18 (6): 2715–54.

Jung, Matthias, Yvonne Schroth, and Andrea Wolf. 2010. "Wählerverhalten und Wahlergebnis." In *Die Bundestagswahl 2009*, edited by Karl-Rodolf Korte, 35–47. New York: Springer.

Kasara, Kimuli, and Pavithra Suryanarayan. 2015. "When Do the Rich Vote Less than the Poor and Why? Explaining Turnout Inequality across the World." *American Journal of Political Science* 59 (3): 613–27.

Lago, Ignacio, José Ramón Montero, and Mariano Torcal. 2007. "The 2006 Regional Election in Catalonia: Exit, Voice, and Electoral Market Failures." *South European Society and Politics* 12 (2): 221–35.

Larcinese, Valentino. 2007. "Voting over Redistribution and the Size of The Welfare State: The Role of Turnout." *Political Studies* 55 (3): 568–85.

Laslier, Jean-François. 2000. "Interpretation of Electoral Mixed Strategies." *Social Choice and Welfare* 17 (2): 283–92.

Ledyard, John O. 1984. "The Pure Theory of Large Two-Candidate Elections." *Public Choice* 44 (1): 7–41.

Lindbeck, Assar, and Jörgen W. Weibull. 1987. "Balanced-Budget Redistribution as the Outcome of Political Competition." *Public Choice* 52 (3): 273–97.

Meltzer, Allan H., and Scott F. Richard. 1981. "A Rational Theory of the Size of Government." *Journal of Political Economy* 89 (5): 914–27.

Roberts, Kevin W. S. 1977. "Voting over Income Tax Schedules." *Journal of Public Economics* 8 (3): 329–40.

Sabet, Navid. 2016. "Turning Out for Redistribution: The Effect of Voter Turnout on Top Marginal Tax Rates." Munich Discussion Paper 2016–13.

Schmidt, Manfred G. 2014. "The Policy of Strategic Demobilisation: The Social Policy of the Christian Democratic-Liberal Coalition, 2009-13." *German Politics* 23 (4): 353–70.

Sheshinski, Eytan. 1972. "The Optimal Linear Income-Tax." *Review of Economic Studies* 39 (3): 297–302.

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